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BACK FLOW FROM JET PLUMES IN VACUUM

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Back Flow From Jet Plumes in Vacuum

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ABSTRACT

Approximate analytical calculations are performed to determine the particle density at surrounding points forward of a jet nozzle in vacuum due to the gas exhausted in the jet plume. The calculations are made for helium, argon and nitrogen. The back flow is determined from the number of particles that leave the continuum regime and cross the particular point in question without experiencing any interparticle collisions.

INTRODUCTION

For spacecraft accelerating or maneuvering in space, the back flow from the jet exhaust may be a large contributor to the ambient environment of the spaceship. In order to get an estimate of the density of the back flow, an approximate calculation is performed for points located in the plane of the nozzle exit.

The gas in the plume is considered to be continuum and Maxwellian until the density decreases to the point where the mean free path is comparable to the distance that the particular mass of gas has travelled from the nozzle exit. Afterwards, the flow is assumed free molecular with no interparticle collisions occuring. The back flow is determined from the number of particles that leave the defined continuum regime and cross the particular point $P(x,y,z)$ in question. This model is depicted in the sketch on the following page.

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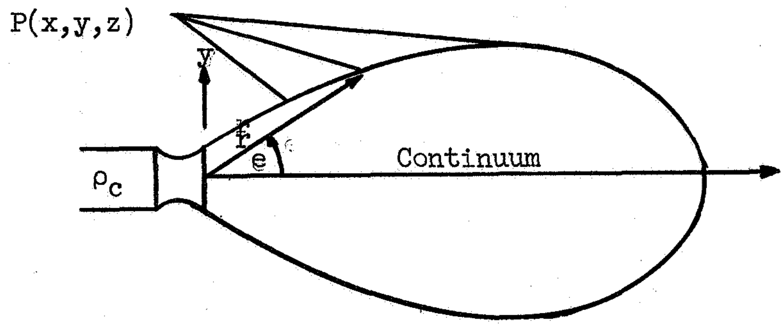

FREE MOLECULAR

Since most of the Plume constant Knudsen surface (defined by Eqs. (5) and (7) below) is usually far from the jet exit plane, it is assumed that the flow has nearly reached its adiabatic limit and has a radial direction; that is, the streamlines diverge from a common source point located at the nozzle exit. The validity of these assumptions is discussed in Refs. 1, 2, and 3. The gases used are helium, nitrogen, and argon. In all the calculations the jet exit Mach number is 5.

ANALYSIS

In Refs. 1, 2, and 3 it is shown that the density along the plume axis of a jet exhausting into a vacuum is approximately given by

$$\rho_{\theta=0} = 4\rho_c B\left(\frac{r^*}{r}\right)^2 \tag{1}$$

where r^* is the nozzle throat radius, r the distance from the source point, ρ_c is the chamber density, and (Ref. 2)

$$B = \frac{\lambda}{4C\sqrt{\pi}}\left(\frac{2}{\gamma + 1}\right)^{1/\gamma-1}\left(\frac{\gamma - 1}{\gamma + 1}\right)^{1/2} \tag{2}$$

where

$$\lambda = \frac{1}{\sqrt{\pi}\left(1 - \frac{C_F}{CC_{F_{max}}}\right)} \tag{3}$$

C_F is the thrust coefficient and C is the ratio of the velocity to the maximum velocity. The ratio, C, is assumed to be approximately 1 everywhere in the plume. The thrust coefficient C_F was evaluated using Eq. (4.33) of Shapiro,[4] γ is the isentropic exponent of the gas.

For r constant, Hill and Draper[2] give the following equation for the density as a function of θ

$$\rho = \rho_{\theta=0}\ e^{-\lambda^2(1-\cos\theta)^2} \tag{4}$$

where λ is given by Eq. (3).

It is assumed that the transition from continuum to free molecular flow occurs when the Knudsen number reaches one. For this study the Knudsen number is defined as

$$K = \frac{\text{mean free path}}{r} = \frac{l}{r} \tag{5}$$

Using $l = 3.736\times10^{-25}\ (M/\rho\delta^2)$ cm., (Ref. 5) where M is the molecular weight and δ is the molecular diameter, we have by solving for ρ

$$\rho = 3.736\times10^{-25}\ \frac{M}{K\delta^2 r},\ \frac{g}{cm^3} \tag{6}$$

Equating (6) to (4) and using (1) for $\rho_{\theta=0}$

$$r = 1.0708\times10^{25}\ \frac{BK\delta^2\rho_c r^{*2}}{M}\ e^{-\lambda^2(1-\cos\theta)^2} \tag{7}$$

which is the equation of the constant Knudsen surface in the plume.

On the constant Knudsen surface the gas is assumed

Maxwellian with a mean velocity $\vec{V}$ in the radial direction. From Noller's[6] paper the density at a point $P(x,y,z)$ due to molecules leaving an elemental volume $d\tau$ adjacent to the constant Knudsen surface is

$$\frac{dn_p}{n_\tau} = \pi^{-3/2}\, e^{-u^2}\left[\frac{1}{2}\, u \cos\psi + \left(\frac{1}{2} + u^2 \cos^2\psi\right) e^{u^2\cos^2\psi}\left(\frac{\sqrt{\pi}}{2}\right)\left(1 + \operatorname{erf}(u\cos\psi)\right)\right] d\Omega \qquad (8)$$

where ψ is the angle between the direction of $\vec{V}$ and the direction of flight of the molecules contributing to the density at $P(x,y,z)$, $u = |\vec{V}|/(2RT/M)$, R the gas constant, T the temperature, and $d\Omega$ is the differential solid angle of $d\tau$ as seen from $P(x,y,z)$. Using the result from another paper[7]

$$d\Omega = \left[\frac{y}{r_e}\sin\theta\cos\varphi\,(1 + \lambda^2\cos\theta - \lambda^2\cos^2\theta) - \frac{r}{r_e} + \frac{z}{r_e}(\cos\theta - \lambda^2\sin^2\theta + \lambda^2\sin^2\theta\cos\theta)\right]\left(\frac{r}{r_e}\right)^2\left(\frac{r_e}{L}\right)^3 \sin\theta\, d\theta\, d\varphi \qquad (9)$$

$$\cos\psi = \left(\frac{r_e}{L}\right)\left(\frac{y}{r_e}\sin\theta\cos\varphi - \frac{r}{r_e} + \frac{z}{re}\cos\theta\right) \qquad (10)$$

$$\left|\frac{L}{r_e}\right|^2 = \left(\frac{y}{r_e}\right)^2 - 2\left(\frac{y}{re}\right)\left(\frac{r}{re}\right)\sin\theta\cos\varphi - 2\left(\frac{r}{r_e}\right)\left(\frac{z}{r_e}\right)\cos\theta + \left(\frac{z}{r_e}\right)^2 + \left(\frac{r}{r_e}\right)^2 \qquad (11)$$

where φ is the angle that the projection of r in the

xy-plane makes with the x-axis. Note that r, θ, φ are the spherical coordinates.

Equation (8) was integrated numerically over the top half of the constant Knudsen number surface. The upper limit for θ was chosen as $\theta_{p,max}$ the maximum Prandtl-Meyer expansion angle from a Mach number of 5. The limits on φ were $\pm\pi/2$. If $r > r_e$ when $\theta = \theta_{p,max}$ the constant Knudsen surface was assumed conical in shape with the cone half-angle being the expansion angle from the exit Mach number of 5 to the Mach number where the density is such that the Knudsen number equals one. For this region

$$d\Omega = \left(\frac{y}{r_e} \cos \theta_p \cos \varphi - \frac{z}{r_e} \sin \theta_p\right)\left(\frac{r_e}{L}\right)\left(\frac{r}{r_e}\right) \sin \theta_p \, d\varphi \, d\left(\frac{r}{r_e}\right) \tag{12}$$

where θ_p is the Prandtl-Meyer expansion angle from Mach 5 to the Mach number such that $K = 1$ and L is the distance from $d\tau$ to point p. From Shapiro

$$\theta_p = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\left[\tan^{-1}\sqrt{\frac{\gamma - 1}{\gamma + 1}(M_K^2 - 1)} - \tan^{-1}\sqrt{\frac{\gamma - 1}{\gamma + 1}(M_e^2 - 1)}\right]$$
$$- \left(\cos^{-1}\frac{1}{M_K} - \cos^{-1}\frac{1}{M_e}\right) \tag{13}$$

where M_e, the exit Mach number, equals 5. M_K is the Mach number where $K = 1$.

$$M_K^2 = \frac{2}{\gamma - 1}\left[\left(\frac{\rho_c}{\rho}\right)^{\gamma-1} - 1\right] \tag{14}$$

For any given $r < r_e$ at $\theta_{p,max}$, ρ is found from Eq. (6) using $K = 1$.

The results for points in a plane located at the nozzle exit are shown in Figs. 1, 2, and 3 for helium, argon, and nitrogen, respectively. Apparent overlapping of curves in some density ranges is not suggested as physically occurring, but is due to changes in assumptions in these ranges.

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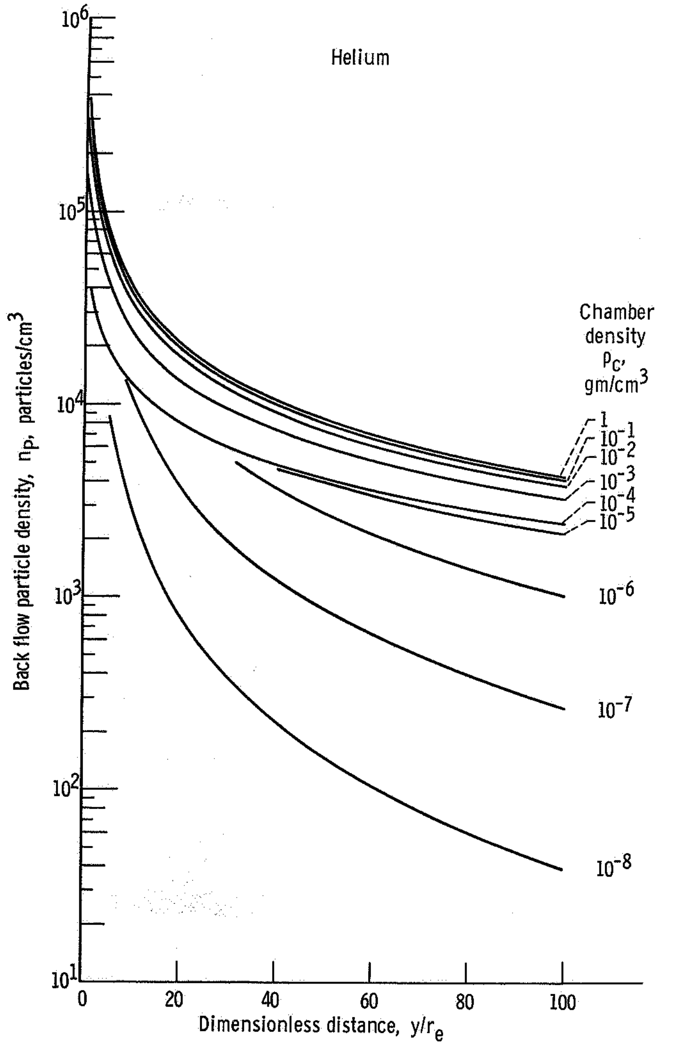

Figure 1.

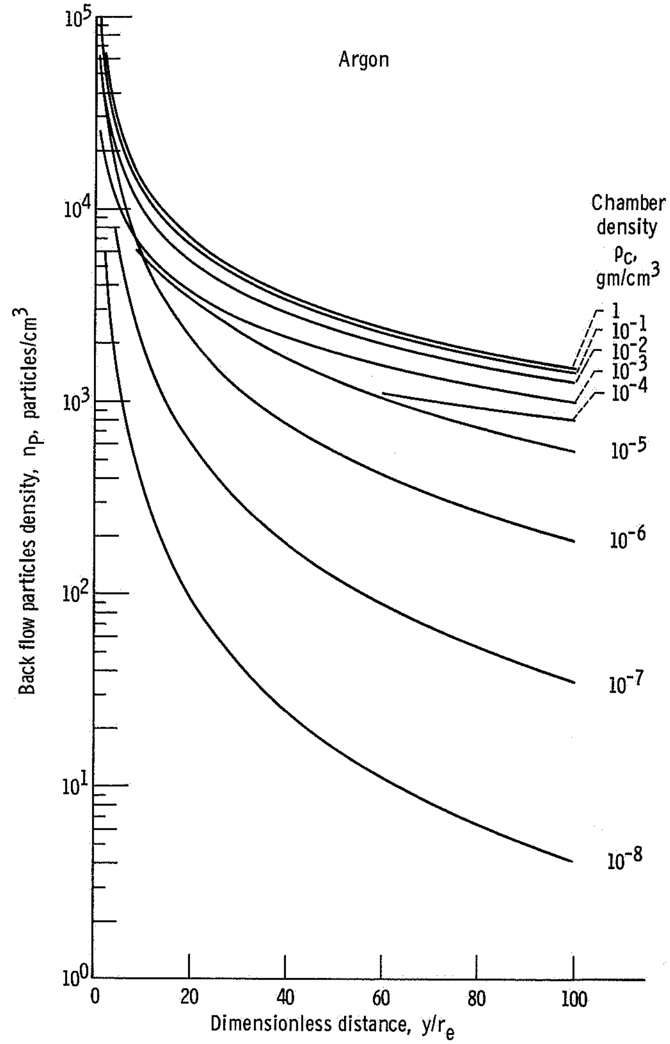

Figure 2.

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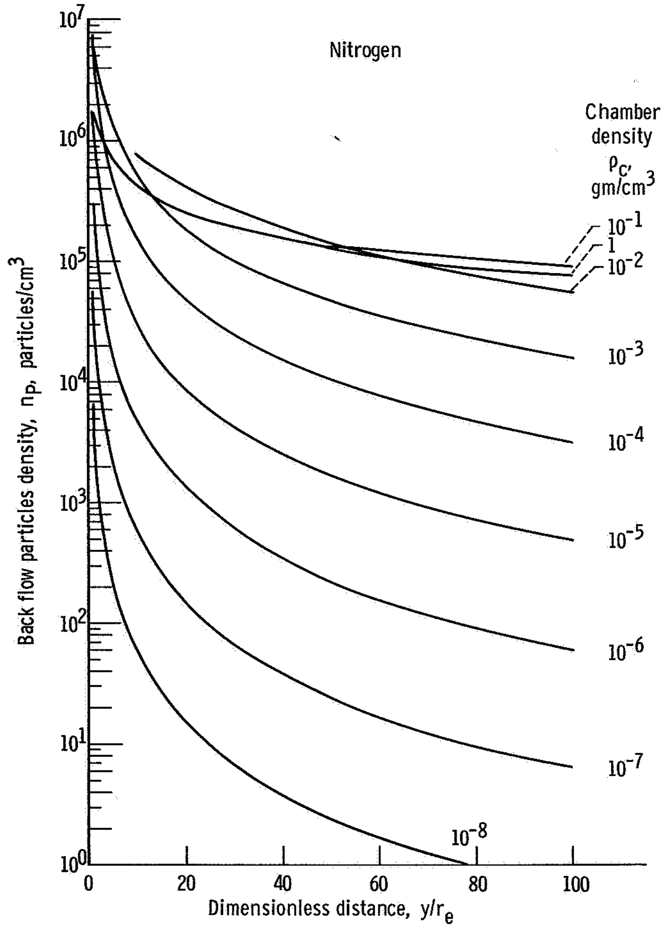

Figure 3.